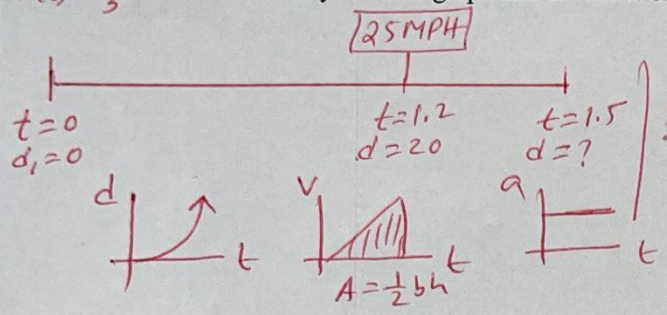
A5: Fazilatunnesa Zoha
Topic: Calculus

**DO Now:**

1. Fazilatunnesa went on to 1.5 hour long bike trip and her position function is $p(t) = \frac{40}{3}t^2$ and in 20 miles of trip she saw a speed limit sign, 25 MPH. Find his final velocity. Did she break the law? Construct a diagram, displacement vs. time graph, velocity vs. time graph and acceleration vs. time graph.

diagram)
$p(t) = \frac{40}{3}t^2$

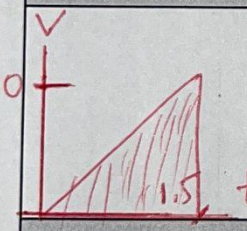


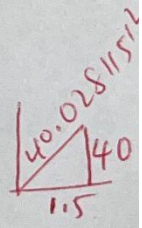

$p(t) = \frac{40}{3}t^2$
$\left(\frac{3}{40}\right)p(t) = \frac{40}{3}t^2\left(\frac{3}{40}\right)$
$t^2 = \sqrt{\frac{3}{2}}$
$t = 1.22$

$p(t) = \frac{40}{3}t^2$
$p'(t) = \frac{80}{3}t^{40}$
$p'(1.5) = \frac{80}{3}\left(\frac{3}{2}\right)$
$V_p = 40$
$p'(t) = \frac{80}{3}$

**Big Idea**

2. The area under the velocity vs. time graph is her total displacement. Construct a VT graph. Shade the area under the curve and find the area (total displacement) seven different ways.

| Construct VT graph | First method | 2nd method | 3rd method |
|---|---|---|---|
|   | $A = \frac{1}{2}bh$ $A = \frac{1}{2}\left(\frac{3}{2}\right)(40)$ $A = (3)(10)$ $A = 30\text{ mile}$ | $V_f^2 = V_i^2 + 2ad$ $40^2 = 0^2 + 2\left(\frac{80}{3}\right)d$ $1600 = \frac{160}{3}d$ $d = 30\text{ mile}$ | $d = v_i t + \frac{1}{2}at^2$ $d = 0(t) + \frac{1}{2}\left(\frac{80}{3}\right)\left(\frac{3}{2}\right)^2$ $d = 30\text{ mile}$ |
| **4th method** | **5th method** | **6th method** (Heron's) | **7th method** |
| $\bar{v} = \frac{v_i + v_f}{2}$ $\bar{v} = \frac{0+40}{2}$ $\bar{v} = 20$ $d = \bar{v}t$ $d = 20\left(\frac{3}{2}\right)$ $d = 30\text{ mile}$ | $p(t) = \frac{40}{3}t^2$ $p\left(\frac{3}{2}\right) = \frac{40}{3}\left(\frac{3}{2}\right)^2$ $= 30\text{ mile}$ | $\sqrt{s(s-a)(s-b)(s-c)}$ $s = 40.764$ $d = \sqrt{40.8(40.8-40.1)(40.8-40)(40.8-1.5)}$ $d = 30\text{miles}$ | $\int_0^{3/2} \frac{80}{3}t\,dt$ $\frac{80}{3}\frac{t^2}{2}\Big\|_0^{\frac{3}{2}}$ $d = 30\text{ mile}$ |




**Exit Slip:**

3. Use the Limit definition of derivative to find instantaneous velocity exactly at the speed limit sign to verify whether she broke the law?

$p(t) = \frac{40}{3}t^2$
$p'(t) = \lim_{\Delta t \to 0} \frac{p(t+\Delta t) - p(t)}{\Delta t}$
$p'(t) = \lim_{\Delta t \to 0} \frac{\frac{40}{3}(t+\Delta t)^2 - \frac{40}{3}t^2}{\Delta t}$
$p'(t) = \lim_{\Delta t \to 0} \frac{\frac{40}{3}(t^2 + 2t\Delta t + \Delta t^2) - \frac{40}{3}t^2}{\Delta t}$

$\frac{40}{3}t^2 + \frac{80}{3}t\Delta t + \frac{40}{3}\Delta t^2 - \frac{40}{3}t^2$
$p'(t) = \lim_{\Delta t \to 0} \frac{\Delta t\left(\frac{80}{3}t + \frac{40}{3}\Delta t\right)}{\Delta t}$
$p'(t) = \lim_{t \to 0} \frac{80}{3}t + \frac{40}{3}\Delta t$
$p'(t) = \frac{80}{3}t$